

On the causal gauge principle

José M. Gracia-Bondía $\dagger \ddagger$

\dagger Departamento de Física Teórica,
Universidad de Zaragoza, Zaragoza 50009, Spain
and

\ddagger Departamento de Física,
Universidad de Costa Rica, San Pedro 2060, Costa Rica

February 12, 2010

Abstract

Work by the Zürich school of causal (Epstein–Glaser) renormalization has shown that renormalizability in the presence of massless or massive gauge fields (as primary entities) explains gauge invariance and, in some instances, the presence of a Higgs-like particle, without need for a Brout–Englert–Higgs–Guralnik–Hagen–Kibble (BEHGHK) mechanism. We review that work, in a pedagogical vein, with a pointer to go beyond.

Contents

1	Introduction	2
2	Overview of the CGI method	3
3	The abelian model	7
3.1	The first-order analysis	7
3.2	The second-order analysis	10
3.3	Higher-order analysis	15
3.4	Summary of the abelian model	16
4	Three MVBs	17

5	The Weinberg–Salam model within CGI	18
5.1	Coupling to matter	19
6	Discussion	21

1 Introduction

By now spontaneous symmetry breaking (SSB) of local symmetry is a well-established paradigm of high-energy physics. At the end of the 60s and beginning of the 70s, it allowed the incorporation of (electro)weak interactions into the framework of renormalizable field theory. In connection with the contemporaneous rise of the Standard Model (SM), it enjoys immense historical success.

However, allusion to unsatisfactory or mysterious aspects of the Higgs sector of the SM does pop up in the literature —see for instance [1, Sect. 22.10]. The Higgs self-coupling terms are completely ad-hoc, unrelated to other aspects of the theory, and do not seem to constitute a gauge interaction. Moreover they raise the hierarchy problem [2, Ch. 11]. The most frequent interpretation of the BEHGHK mechanism clashes with cosmology [3].

Debate on the proper interpretation of the mechanism (whether the symmetry is “broken” or just “hidden”, whether the Higgs field truly has a non-zero vacuum expectation value (VEV) or not [4], and so on) seems endless. This breeds some skepticism, even among earlier and doughty practitioners. At the end of his Nobel lecture [5], Veltman chose to declare: “*While theoretically the use of spontaneous symmetry breakdown leads to renormalizable Lagrangians, the question of whether this is really what happens in Nature is entirely open*”.

Indeed, since the *deus ex machina* fields involved in broken or hidden symmetry are unobservable, the status question for the BEHGHK contraption cannot be resolved by the likely sighting of the Higgs particle in the LHC.¹

The subject has also been obscured all along by theoretical prejudice. In the SM the Higgs field carries the load of giving masses to *all* matter and force fields. For instance, it is said that mass terms for the vector bosons are incompatible with gauge invariance. It ain’t so: such mass terms fit in gauge theory by use of Stückelberg fields [8, 9].

Skepticism would be idle, nevertheless, in the absence of alternative theoretical frameworks. Assuming an agnostic stance, we pose the question: is

¹This situation has recently called the attention of knowledgeable philosophers of science [6, 7]: in epistemological terms, they argue that the mechanism had heuristic value in the context of discovery; but much less so in the context of justification.

it possible to formulate the main results of flavourdynamics, and to frame suggestions of new physics, without recourse to unobservable processes? In tune with the phenomenological SM Lagrangian [10], this amounts to regard massive vector bosons (MVB) as fundamental entities.

So let us stop pretending we know the origins of mass. Higgs-like scalar fields will still come in handy for either renormalizability or unitarity; however, their gauge variations need not be the conventional ones. Fermions can be assigned Dirac masses, and couplings with the scalar field proportional to those; this contradicts in no way the chiral nature of their interactions in the SM.

An approach with the mentioned traits is already found in the literature in the work by Scharf, Dütsch and others, under the label of the “quantum gauge invariance” principle. A few references to it are [11–14] and mainly the book [15]. The “quantum Noether principle” of [16, 17] coincides essentially with it. Both are based on the rigorous causal scheme for renormalization [18] by Epstein and Glaser (EG).

Henceforth we refer to the approach as causal gauge invariance (CGI). The usual plan of the article is found at the end of the next section, when the stakes hopefully have been made clearer.

2 Overview of the CGI method

The spirit of CGI is very much that of the [19]. Let s denote the nilpotent BRS operation. To realize gauge symmetry, one should incorporate BRS symmetry ab initio in a “quantum” Lagrangian \mathcal{L} , such that (very roughly speaking) $s\mathcal{L} \sim 0$, and proceed to build from there. We do this for MVBs.

The starting point for the analysis is the Bogoliubov–Epstein–Glaser functional scattering matrix on Fock space, in the form of a power series:

$$\mathbb{S}(g) = 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!} \int dx_1 \dots dx_n T_n(x_1, \dots, x_n) g(x_1) \dots g(x_n). \quad (1)$$

The coupling constants of the model are replaced by test functions —we wrote just one of them for simplicity. The theory is then constructed basically by using causality and Poincaré invariance to recursively determine the form of the time-ordered products T_n from the T_m with $m < n$; in this sense the procedure is inverse to the “cutting rules”. Only those fields should appear in T_n that already are present in T_1 . The procedure yields a finite perturbation theory without regularization; ultraviolet divergences are avoided by proper definition of the n -point functions as distributions.

(Ultimately one would be interested in the adiabatic limit $g(x) \uparrow g$. This is delicate, however, due to infrared problems. We look at the theory before that limit is taken.)

With the proviso that two forms of T_n are equivalent if they differ by s -coboundaries, CGI is formulated by the fact that sT_n must be a divergence. Roughly speaking, we must have

$$\begin{aligned} sT_n(x_1, \dots, x_n) &= i \sum_{l=1}^{l=n} T[T_1(x_1), \dots, \partial_l \cdot Q(x_l), \dots, T_1(x_n)], \\ &=: i \sum_{l=1}^{l=n} \partial_l \cdot Q_n(x_1, \dots, x_n). \end{aligned} \quad (2)$$

for vectors Q_n , called Q -vertices, with ∂_l denoting the partial divergences with respect to the x_l coordinates and T a time-ordering operator. In this way renormalization and gauge invariance are linked in the EG scheme. (We said “roughly” because (2) suggests that T and spacetime derivatives commute, which is not generally the case for on-shell fields.)

Note that T_1 only contains the first-order part of the Lagrangian. Nevertheless, already the first order condition

$$sT_1 = i\partial \cdot Q_1$$

constrains significantly the form of the Lagrangian. Later on, we show leisurely how the CGI method works for tree graphs belonging to T_2 . This is almost all what is required for the purposes of this paper: for ordinary gauge theories, the treatment of T_3 is pretty simple, and higher orders not needed at all.

Keep in mind that one works here with *free* fields. Interacting fields can be arrived at in the Epstein–Glaser procedure, somewhat a posteriori, using their definition by Bogoliubov as logarithmic functional derivatives of $\mathbb{S}(g)$ with respect to appropriate sources. Their gauge variations resemble more those of standard treatments; but we do not use them. Thus s “sees” only the (massive or massless) gauge fields, and the attending (anti-)ghost and Stückelberg fields. This is why everything flows from the quantum gauge structure of the boson sector. Coupling to fermions, which ought not be organized in multiplets a priori, comes almost like an afterthought.

As it turns out, the procedure is quite restrictive, and in particular only a few models for MVB theory pass muster. These exhibit very definite mass and interaction patterns, in particular quartic self-interaction for the scalar particles.

We next compile the results, according to [15]. Consider a model with t intermediate vector bosons A_a in all, of which any may be in principle massive or massless. Let us say there are r massive ones with masses m_a , $1 \leq a \leq r$ and s massless ones, and $t = r + s$. We assume there is *one* (at most) physical scalar particle H of mass m_H : *entia non sunt multiplicanda praeter necessitatem*. The BRS extension of the Wigner representation theory for MVBs requires Stückelberg fields B_a [13], beyond the fermionic ghosts u_a, \tilde{u}_a ; in case A_a is massless, we of course let B_a drop out. Adopting the Feynman gauge, the gauge variations are as follows:

$$\begin{aligned} sA_a^\mu(x) &= i\partial^\mu u_a(x); \\ sB_a(x) &= im_a u_a(x); \\ su_a(x) &= 0; \\ s\tilde{u}_a(x) &= -i(\partial \cdot A_a(x) + m_a B_a(x)). \\ sH(x) &= 0. \end{aligned} \tag{3}$$

This operator is nilpotent on-shell.

The total bosonic interaction Lagrangian, in a notation close to that of [15], is of the form

$$\mathcal{L}_{\text{int}} = gT_1 + \frac{g^2 T_2}{2}, \tag{4}$$

where g is an overall dimensionless coupling constant;

$$T_1 = f_{abc}(T_{1abc}^1 + T_{1abc}^2 + T_{1abc}^3 + T_{1abc}^4) + C(T_1^5 + T_1^6 + T_1^7 + T_1^8 + T_1^9)$$

includes the cubic couplings, and

$$T_2 = T_2^1 + T_2^2 + T_2^3 + T_2^4 + T_2^5 + T_2^6 + T_2^7$$

includes the quartic ones. The list of cubic couplings not involving H is given by:

$$\begin{aligned} T_{1abc}^1 &= [A_a \cdot (A_b \cdot \partial) A_c - u_b(A_a \cdot \partial \tilde{u}_c)]; \\ T_{1abc}^2 &= \frac{m_b^2 + m_c^2 - m_a^2}{4m_b m_c} [B_b(A_a \cdot \partial B_c) - B_c(A_a \cdot \partial B_b)]; \\ T_{1abc}^3 &= \frac{m_b^2 - m_a^2}{2m_c} (A_a \cdot A_b) B_c; \\ T_{1abc}^4 &= \frac{m_a^2 + m_c^2 - m_b^2}{2m_c} \tilde{u}_a u_b B_c; \end{aligned} \tag{5}$$

The list of cubic couplings of the Higgs-like particle is:

$$\begin{aligned}
T_1^5 &= m_a [B_a (A_a \cdot \partial H) - H (A_a \cdot \partial B_a)]; \\
T_1^6 &= m_a^2 (A_a \cdot A_a) H; \\
T_1^7 &= -m_a^2 \tilde{u}_a u_a H; \\
T_1^8 &= -\frac{1}{2} m_H^2 B_a^2 H; \\
T_1^9 &= -\frac{1}{2} m_H^2 H^3.
\end{aligned} \tag{6}$$

Remarks: in (5) and (6) we sum over repeated indices; the f_{abc} are completely skewsymmetric in their three indices, and fulfil the Jacobi identity; T_1^1 yields the cubic part in the classical Yang–Mills Lagrangian; C is a constant independent of a . The dimension of the Lagrangian must be M^4 in natural units, and the boson field dimension in our formulation is 1 for *both* spins: the dimension of C is M^{-1} . Note the diagonality of the couplings of the Higgs-like particle. Crossed terms like $(A_a \cdot A_b)H$ for $a \neq b$, and others like $B_a B_b B_c$, $B_a H^2 \dots$, that could be envisaged, are held to vanish by CGI.

The list of quartic couplings:

$$\begin{aligned}
T_2^1 &= -\frac{1}{2} f_{abc} f_{ade} (A_b \cdot A_d) (A_c \cdot A_e); \\
T_2^2 &= \left[\frac{(m_d^2 + m_e^2 - m_a^2)(m_c^2 + m_e^2 - m_b^2)}{8m_d m_c m_e^2} f_{ade} f_{bce} + c \leftrightarrow d \right. \\
&\quad \left. + \frac{1}{2} C^2 m_a m_b \delta_{ad} \delta_{bc} + c \leftrightarrow d \right] \times (A_a \cdot A_b) B_c B_d; \\
T_2^3 &= -\frac{1}{4} C^2 m_H^2 B_a^2 B_b^2 \quad \text{irrespective of } a, b \leq r; \\
T_2^4 &= C f_{abc} \frac{m_b^2 - m_a^2}{m_c} (A_a \cdot A_b) B_c H; \\
T_2^5 &= C^2 m_a^2 (A_a \cdot A_a) H^2; \\
T_2^6 &= -\frac{1}{2} C^2 m_H^2 B_a^2 H^2 \quad \text{irrespective of } a \leq r; \\
T_2^7 &= -\frac{1}{4} C^2 m_H^2 H^4.
\end{aligned} \tag{7}$$

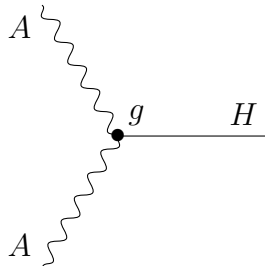
Every coefficient of the interaction Lagrangian is in principle determined in terms of the f_{abc} and the pattern of masses. We are not through, because CGI implies *constraints*, in general non-linear and extremely restrictive, on *allowed patterns* of masses for the gauge fields. But we may anticipate a few more comments. The first term T_2^1 in (7) just yields the quartic part in the classical Yang–Mills Lagrangian, as expected. In case all the A_a are massless, there is no need to add physical or unphysical scalar fields for renormalizability, and only T_1^1 and T_2^1 survive in the theory; they of course coincide respectively with the first and second order part of the usual Yang–Mills Lagrangian. In particular, CGI gives rise to gluodynamics. (It must

be said, though, that the physical equivalence of couplings differing in a divergence is less compelling in this case, since there is no asymptotic limit for the Bogoliubov–Epstein–Glaser $\mathbb{S}(g)$ -matrix; CGI offers no tools to deal with this infrared problem.) Remarkably, with independence of the masses, CGI unambiguously leads to generalized Yang–Mills theories on reductive Lie algebras; apparently this was realized first by Stora [20].

The plan of the rest of the article is as follows. Notice that the case $r = 1, s = 0$ leads to an abelian model in which all the terms with the Higgs-like field H survive. We use this example in Section 3 to illustrate in some detail—missing in [15]—how the second-order condition determines the couplings. Section 4 deals with *three* gauge fields—there are no models with two gauge fields to speak of, since $\mathfrak{u}(1) \oplus \mathfrak{u}(1)$ is the only two-dimensional reductive Lie algebra. For that we need to invoke the mentioned mass relations (reference [15] unfortunately contains misprints in this respect). Section 5 elaborates on the reconstruction of the SM in CGI, looking at the fermion sector as well. The paper ends with a discussion.

3 The abelian model

Consider a theory with a neutral gauge field A of mass m and a *physical* neutral scalar field H of mass m_H , and basic coupling AAH .



Since massive quantum electrodynamics is known to be renormalizable without an extra scalar field, this is perhaps not very interesting; but our aim here is merely showing the workings of the causal gauge principle.

3.1 The first-order analysis

For T_1 , take the most general Ansatz containing cubic terms in the fields and leading to a renormalizable theory. With the benefit of hindsight, we write

down on the first line the terms destined to survive:

$$T_1/m = (A \cdot A)H + b\tilde{u}uH + c(H(A \cdot \partial B) - B(A \cdot \partial H)) + dB^2H + eH^3 \\ + a(A \cdot A)B + b_2\tilde{u}uB + b_3u(A \cdot \partial \tilde{u}) + d_1B^3 + d_3BH^2. \quad (8)$$

The factor m is natural according to our previous discussion on dimensions. The symmetric combination $HA \cdot \partial B + BA \cdot \partial H$ has been excluded for the following reason:

$$A \cdot (B\partial H + H\partial B) = \partial \cdot (BHA) - (\partial \cdot A)BH,$$

and in view of (3), the $(\partial \cdot A)BH$ term is s -exact apart from terms of already present in (8). Concretely,

$$s(\tilde{u}BH) = -(\partial \cdot A)BH - mB^2H - m\tilde{u}uH.$$

We calculate next sT_1/m in (8) and obtain for the first group of terms:

$$2\partial \cdot (uHA) - 2u(\partial \cdot A)H - 2uA \cdot \partial H - bu(\partial \cdot A)H \\ - bmuBH + c\partial \cdot u(H\partial B - B\partial H) \\ + cm[HA \cdot (\partial u) - uA \cdot \partial H] + 2dmuBH. \quad (9)$$

We have used $s(uC) = -usC$ for any C . In detail:

$$-is(A \cdot AH) = (\partial u \cdot A)H = 2[\partial \cdot (uHA) - u(\partial \cdot A)H - uA \cdot \partial \cdot H].$$

Next

$$-is(\tilde{u}uH) = -u(\partial \cdot A)H - muBH.$$

Next

$$-is(A \cdot (H\partial B - B\partial H)) = \partial u \cdot (H\partial B - B\partial H) + m[HA \cdot (\partial u) - uA \cdot \partial H].$$

Finally $-is(B^2H) = 2muBH$.

Similarly, for the second group of terms we obtain:

$$2a\partial \cdot (uBA) - 2au(\partial \cdot A)B - 2auA \cdot \partial B + amuA \cdot A \\ - b_2u(\partial \cdot A)B - b_2muB^2 + b_3(\partial u \cdot u \partial \tilde{u} + uA \cdot \partial(\partial \cdot A + mB)) \\ + 3d_1muB^2 + d_3muH^2.$$

All terms of that group are excluded because their contributions to sT_1 are not pure divergences. For instance, the first one corresponds to the term in $uA \cdot A$, that can be canceled only by setting $a = 0$.

On the other hand, the second term in the second line in (9) can be recast as

$$\partial \cdot (u(H\partial B - B\partial H)) + (m^2 - m_H^2)uBH.$$

For the following terms we have

$$A \cdot (\partial u)H - uA \cdot \partial H = \partial \cdot (uHA) - u(\partial \cdot A)H - 2uA \cdot \partial H.$$

In all,

$$\begin{aligned} -isT_1/m &= \partial \cdot (C + D) - (2 + cm + b)u(\partial \cdot A)H \\ &- (2 + 2cm)u(A \cdot \partial H) + (2dm - bm + c(m^2 - m_H^2))uBH; \end{aligned}$$

with the vectors C, D given by $C := (2 + cm)uHA$; $D := cu(H\partial B - B\partial H)$. The terms that are not a divergence must cancel. This at once leads to:

$$c = -\frac{1}{m}; \quad b = -1; \quad d = -\frac{m_H^2}{2m^2}; \quad \text{thus} \quad C = uHA; \quad D = \frac{-u}{m}(H\partial B - B\partial H).$$

In summary, we have obtained the cubic couplings in the Lagrangian:

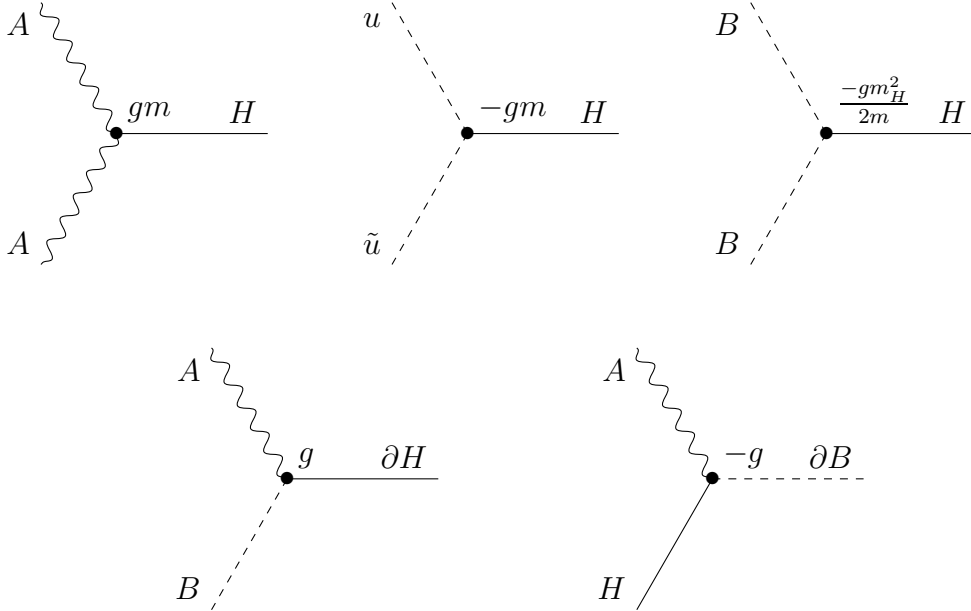
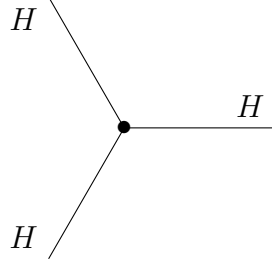


Figure 1: cubic vertices

To this we should add the H^3 coupling, whose coefficient is still indeterminate:



Moreover:

$$sT_1 = i\partial \cdot Q_1 \quad \text{with} \quad Q_1 = muHA - u(H\partial B - B\partial H). \quad (10)$$

3.2 The second-order analysis

The next step is less trivial. Equation (2) certainly makes sense outside the diagonals, for then the T product is calculated like an ordinary product. But the extension to the diagonals, which is simply $x_1 = x_2$ for $n = 2$, can produce local correction terms. At this order, the advanced and retarded products are given by:

$$\begin{aligned} A_2(x_1, x_2) &= T_2(x_1, x_2) - T_1(x_1)T_1(x_2); \\ R_2(x_1, x_2) &= T_2(x_1, x_2) - T_1(x_2)T_1(x_1); \end{aligned} \quad (11)$$

Here $T_2(x_1, x_2)$ is still unknown, but it is clear that A_2 will have support on the past light cone of x_2 , and R_2 on its future light cone; hence the nomenclature. Consider then $D_2(x, y) := (R_2 - A_2)(x, y) = [T_1(x), T_1(y)]$, whose support is within the light cone (we say D_2 is causal). We have thus

$$\begin{aligned} sD_2(x, y) &= [sT_1(x), T_1(y)] + [T_1(x), sT_1(y)] \\ &= i\partial_x[Q_1(x), T_1(y)] + i\partial_y[T_1(x), Q_1(y)]; \end{aligned} \quad (12)$$

so that D_2 moreover is gauge-invariant. The crucial step in EG renormalization is the *splitting* of D_2 into the retarded part R_2 and the advanced part A_2 ; once this is done, T_2 is found at once from (11). The issue is how to preserve gauge invariance in this distribution splitting. For this, we split D_2 and the commutators —without the derivatives— in the previous equation; then gauge invariance:

$$sR_2(x, y) = i\partial_x R_{2/1}(x, y) + i\partial_y R_{2/2}(x, y)$$

can only be (and is) violated for $x = y$, that is, by local terms in $\delta(x - y)$. However, if in turn local renormalization terms $N_2, N_{2/1}, N_{2/2}$ can be found in such a way that

$$s(R_2(x, y) + N_2(x, y)) = i\partial_x(R_{2/1} + N_{2/1}) + i\partial_y(R_{2/2} + N_{2/2}),$$

with an obvious notation, then CGI to second order holds.

To the purpose we consider only tree diagrams. In view of (12), we systematically proceed to study the divergences coming from cross-terms between (10) and

$$T_1 = m[(A \cdot A)H + u\tilde{u}H - \frac{1}{m}A \cdot (H\partial B - B\partial H) - \frac{m_H^2}{2m^2}B^2H + eH^3]. \quad (13)$$

Factors containing derivatives give rise to normalization contributions after distribution splitting.

The most difficult part of the coming calculation asks for divergences of terms with commutators $[\partial^\mu B(x), \partial^\nu B(y)]$ and $[\partial^\mu H(x), \partial^\nu H(y)]$. Following [21], we look at Section 4 in [11] in order to prepare the computation. There, for general functions F, E we find the formulas:

$$\begin{aligned} & \partial_\mu^x[F(x)E(y)\delta(x-y)] + \partial_\mu^y[F(y)E(x)\delta(x-y)] \\ &= \partial_\mu F(x) E(x)\delta(x-y) + F(x) \partial_\mu E(x)\delta(x-y) \end{aligned} \quad (14)$$

$$\begin{aligned} \text{and } & F(x)E(y)\partial_\mu^x\delta(x-y) + F(y)E(x)\partial_\mu^y\delta(x-y) \\ &= F(x) \partial_\mu E(x)\delta(x-y) - \partial_\mu F(x) E(x)\delta(x-y). \end{aligned} \quad (15)$$

We may prove both from the following observation: since

$$F(x)E(y)\delta(x-y) = F(x)E(x)\delta(x-y),$$

it must be that

$$\partial_\mu^x(F(x)E(y)\delta(x-y)) = \partial_\mu^x(F(x)E(x)\delta(x-y));$$

which forces

$$E(y)\partial_\mu^x\delta(x-y) = E(x)\partial_\mu^x\delta(x-y) + \partial_\mu E(x)\delta(x-y). \quad (16)$$

Now,

$$\begin{aligned} & \partial_\mu^x[F(x)E(y)\delta(x-y)] + \partial_\mu^y[F(y)E(x)\delta(x-y)] \\ &= \partial_\mu F(x) E(x)\delta(x-y) + F(x)E(y)\partial_\mu^x\delta(x-y) \\ &+ \partial_\mu F(x) E(x)\delta(x-y) - F(y)E(x)\partial_\mu^x\delta(x-y) \\ &= \partial_\mu F(x) E(x)\delta(x-y) + F(x)E(y)\partial_\mu^x\delta(x-y) \\ &- F(x)E(x)\partial_\mu^x\delta(x-y) = \partial_\mu F(x) E(x)\delta(x-y) + F(x) \partial_\mu E(x)\delta(x-y); \end{aligned}$$

where we have used (16) twice. Analogously,

$$\begin{aligned} F(x)E(y)\partial_\mu^x\delta(x-y) + F(y)E(x)\partial_\mu^y\delta(x-y) &= F(x)E(x)\partial_\mu^x\delta(x-y) \\ &+ F(x)\partial_\mu E(x)\delta(x-y) - F(y)E(x)\partial_\mu^x\delta(x-y) \\ &= F(x)\partial_\mu E(x)\delta(x-y) - \partial_\mu F(x)E(x)\delta(x-y), \end{aligned}$$

using (16) twice again.

We finally start the advertised computation. Coming from respectively the second term of $Q_1(x)$ in (10) and third of $T_1(y)$ in (13), now we find for $i[Q_1(x), T_1(y)]$:

$$\begin{aligned} iu(x)H(x)[\partial^\mu B(x), \partial^\nu B(y)]A_\nu(y)H(y) \\ = u(x)H(x)A_\nu(y)H(y)\partial_x^\mu\partial_y^\nu D(x-y). \end{aligned}$$

The identity $[B(x), B(y)] = -iD(x-y)$ for scalar fields has been employed. Next we need to tackle the divergence of the splitting of $\partial_x^\mu\partial_y^\alpha D$. Splitting of the Jordan–Pauli propagator D gives rise to the retarded propagator D^{ret} . Now, each derivation increases by one the singular order of a distribution. Thus, although $\partial_x^\mu\partial_y^\nu D^{\text{ret}}$ is a well-defined distribution, its singular order is $-2+2=0$, therefore allowing a normalization term in the split distribution:

$$\partial_x^\mu\partial_y^\nu D^{\text{ret}}(x-y) \rightarrow \partial_x^\mu\partial_y^\nu D^{\text{ret}}(x-y) + C_B g^{\mu\nu}\delta(x-y).$$

After applying ∂_μ , simply from

$$\partial_\mu^x\partial_x^\mu D^{\text{ret}}(x-y) = -m^2 D^{\text{ret}}(x-y) + \delta(x-y),$$

the total singular part is of the form

$$C_B\partial_x^\nu[F(x)E(y)\delta(x-y)] + F(x)E(y)\partial_y^\nu\delta(x-y), \quad \text{with } F = uH; E = HA_\nu.$$

Adding the term with x and y interchanged, and using the identities (14) and (15), it comes finally the short rule for this kind of singular term:

$$F(x)E(y)\partial_x^\mu\partial_y^\nu D(x-y) \rightarrow [(C_B + 1)(\partial^\nu F)E + (C_B - 1)F\partial^\nu E]\delta(x-y).$$

Therefore we obtain in the end

$$\begin{aligned} (C_B + 1)[H^2(A \cdot \partial u) + uH(A \cdot \partial H)]\delta(x-y) \\ + (C_B - 1)[uH^2(\partial \cdot A) + uH(A \cdot \partial H)]\delta(x-y). \end{aligned} \tag{17}$$

By the same token, coming now from respectively the third and fourth terms in $Q_1(x)$ and $T_1(y)$, and performing entirely similar operations, we obtain

$$(C_H + 1)[B^2(A \cdot \partial u) + uB(A \cdot \partial B)]\delta(x - y) \\ + (C_H - 1)[uB^2(\partial \cdot A) + uB(A \cdot \partial B)]\delta(x - y). \quad (18)$$

There is no good reason for $C_H \neq C_B$; see further on.

There are no singular contributions from the first term in $Q_1(x)$. The second term there will contribute for the commutators with the fourth and fifth terms in $T_1(y)$. Concretely, there is the term

$$-iu(x)H(x)[\partial^\mu B(x), B(y)]A_\nu(y)\partial^\nu H(y) \\ = -u(x)H(x)A_\nu(y)\partial^\nu H(y)\partial_x^\mu D(x - y),$$

plus the analogous one in $[T_1(x), Q_1(y)]$. We are led to the singular part

$$-2uH(A \cdot \partial H)\delta(x - y). \quad (19)$$

The short rule here is $\partial^\mu D \rightarrow 2\delta$.

Next, we obtain

$$\frac{im_H^2}{m}u(x)H(x)[\partial^\mu B(x), B(y)]B(y)H(y) \\ = \frac{m_H^2}{m}u(x)H(x)B(y)H(y)\partial_x^\mu D(x - y),$$

leading to the singular part

$$\frac{2m_H^2}{m}uBH^2\delta(x - y). \quad (20)$$

From the last term in Q_1 , combining with the first term in $T_1(y)$, we obtain in all the singular part

$$2muB(A \cdot A)\delta(x - y). \quad (21)$$

Combining both third terms, we consider

$$-iu(x)B(x)[\partial^\mu H(x), H(y)]A_\nu(y)\partial^\nu B(y) \\ = -u(x)B(x)A_\nu(y)\partial^\nu B(y)\partial_x^\mu D_{m_H}(x - y).$$

We have in all the singular part:

$$-2uB(A \cdot \partial B)\delta(x - y). \quad (22)$$

Coming from respectively the third term in Q and the fifth term in T_1 , there is the commutator

$$\begin{aligned} & \frac{-im_H^2}{2m} u(x) B(x) [\partial^\mu H(x), H(y)] B^2(y) \\ &= -\frac{m_H^2}{m} u(x) B(x) B^2(y) \partial_x^\mu D_{m_H}(x-y). \end{aligned}$$

After collecting the similar term and taking the divergences, this leads to

$$-\frac{m_H^2}{m^3} u B^3 \delta(x-y). \quad (23)$$

Coming respectively from the third and sixth term, there is the commutator

$$\begin{aligned} & 3imeu(x) B(x) [\partial^\mu H(x), H(y)] H^2(y) \\ &= 3emu(x) B(x) H^2(y) \partial_x^\mu D_{m_H}(x-y). \end{aligned}$$

After taking the divergences, this leads to a total singular part

$$6emu B H^2 \delta(x-y). \quad (24)$$

Next we list all possible normalization terms. Among them, the two first ones are coming from second-order tree graphs with two derivatives on the inner line. In other words, they come from $s[T_1(x), T_1(y)]$. Indeed, in this causal distribution, combining the third terms in the expression of T_1 , there appears the term

$$\begin{aligned} & iA_\mu(x) H(x) [\partial^\mu B(x), \partial^\nu B(y)] A_\nu(y) H(y) \\ &= A_\mu(x) H(x) A_\nu(y) H(y) \partial_x^\mu \partial_x^\nu D(x-y). \end{aligned}$$

This leads us to a normalization term $C_B(A \cdot A) H^2 \delta(x-y)$. By the same token, the reader may verify that combining the fourth terms in the expression of T_1 there appears the normalization term $C_H(A \cdot A) B^2 \delta(x-y)$.

However, any term of the same form, compatible with Poincaré covariance, discrete symmetries, ghost number and power counting represents in principle a legitimate normalization. Thus we introduce the list of (re)normalization terms we need:

$$\begin{aligned} N_2^1 &= C_B(A \cdot A) H^2 \delta(x-y); \\ N_2^2 &= C_H(A \cdot A) B^2 \delta(x-y); \\ N_2^3 &= -\frac{m_H^2}{4m^2} B^4 \delta(x-y); \\ N_2^4 &= \left(\frac{m_H^2}{m^2} + 3e \right) B^2 H^2 \delta(x-y). \end{aligned}$$

In view of (1) they generate new couplings. There is also a N_2^5 term in H^4 , that we omit for now. For convenience, we have anticipated the coefficients in N_2^3, N_2^4 , which are of the second class. The normalization terms amount to new vertices with four external legs. We compute the coboundaries:

$$\begin{aligned} sN_2^1 &= 2C_B H^2 (A \cdot \partial u) \delta(x - y); \\ sN_2^2 &= 2C_H [B^2 (A \cdot \partial u) + muB(A \cdot A)] \delta(x - y); \\ sN_2^3 &= -\frac{m_H^2}{m} u B^3 \delta(x - y); \\ sN_2^4 &= \left(\frac{2m_H^2}{m} + 6em \right) u B H^2 \delta(x - y). \end{aligned}$$

The cancellation now is easy to obtain: let $C_B = C_H = 1$. This means that we have only to worry about the first two terms in (17) and similarly in (18). Now, respectively the term (19) cancels the second one in (17) and the term (22) cancels the second one in (18). The two remaining terms in (17) and (18), together with (20), (21), (23) and (24) are exactly accounted for thanks to the normalization summands.

Therefore we have determined T_1 and T_2 , except that e still remains indeterminate. But please read on.

3.3 Higher-order analysis

For the higher-order analysis, it is convenient to have the expansion of the inverse \mathbb{S} -matrix:

$$\mathbb{S}^{-1}(g) =: 1 + \sum_1^\infty \frac{i^n}{n!} \int d^4 x_1 \dots \int d^4 x_n \bar{T}_n(x_1, \dots, x_n) g(x_1) \dots g(x_n).$$

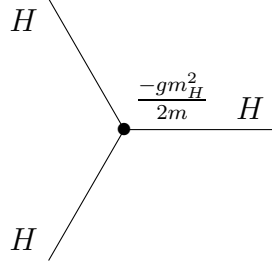
For instance, the second order term $\bar{T}_2(x_1, x_2)$ in the expansion of $\mathbb{S}^{-1}(g)$ is given by

$$\bar{T}_2(x_1, x_2) = -T_2(x_1, x_2) + T_1(x_1)T_1(x_2) + T_1(x_2)T_1(x_1).$$

Then, say,

$$\begin{aligned} A_3(x_1, x_2, x_3) &= \bar{T}_1(x_1)T_2(x_2, x_3) + \bar{T}_1(x_2)T_2(x_1, x_3) + \bar{T}_2(x_1, x_2)T_1(x_3) \\ &\quad + T_3(x_1, x_2, x_3); \\ R_3(x_1, x_2, x_3) &= T_1(x_3)\bar{T}_2(x_1, x_2) + T_2(x_1, x_3)\bar{T}_1(x_2) + T_2(x_2, x_3)\bar{T}_2(x_1) \\ &\quad + T_3(x_1, x_2, x_3). \end{aligned}$$

Just as before, $D_3 := R_3 - A_3$ depends only on known quantities, is causal in x_3 and is gauge invariant. Splitting it, we can calculate T_3 . We refer to [15] for the outcome of the analysis in our case, which turns out to be quite simple. The missing cubic term is given by:



Also, it is seen that we need the new normalization term $N_2^5 = fH^4\delta$. One finds $f = -m_H^2/4m^2$, and we are home.

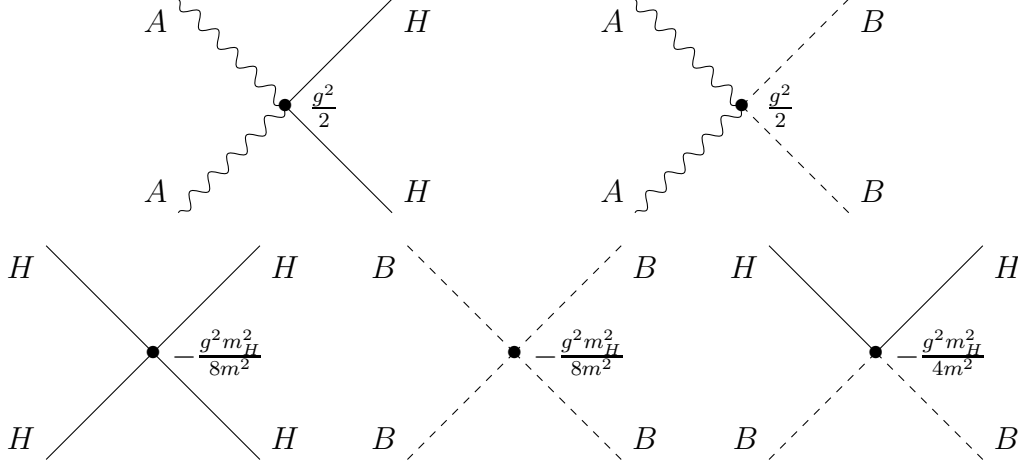
3.4 Summary of the abelian model

Thus we write down the final (interaction) Lagrangian associated to the abelian theory of the previous section. There are two physical fields A^μ, H , of respective masses m, m_H , and an assortment of ghosts u, \tilde{u}, B , which in our Feynman gauge all possess mass m . We obtained six cubic couplings (proportional to g) and five quartic ones (proportional to g^2). It is remarkable that CGI generates the latter from the former. Only four terms out of the eleven involve couplings exclusively among the physical fields.

$$\begin{aligned} \mathcal{L}_{\text{int}}(x) = & gm(A \cdot A)H - gm\tilde{u}uH + gB(A \cdot \partial H) \\ & - gH(A \cdot \partial B) - \frac{gm_H^2}{2m}H^3 - \frac{gm_H^2}{2m}B^2H \\ & + \frac{g^2}{2}(A \cdot A)H^2 + \frac{g^2}{2}(A \cdot A)B^2 \\ & - \frac{g^2m_H^2}{8m^2}H^4 - \frac{g^2m_H^2}{4m^2}H^2B^2 - \frac{g^2m_H^2}{8m^2}B^4. \end{aligned}$$

With $C = 1/m$ this tails down perfectly with (4) together with (5), (6) and (7). By construction the total Lagrangian is BRS invariant in the sense defined here. (It has been proved recently in a rigorous way [22] in the EG framework for interacting fields that “classical” BRS invariance implies gauge invariance for all tree graphs at all orders.)

We exhibit the quartic interaction vertices graphically.



Notice that the purely scalar couplings are

$$\begin{aligned}
& -g \frac{m_H^2}{2m} H(B^2 + H^2) - g^2 \frac{m_H^2}{8m^2} (B^2 + H^2)^2 \\
& = -\frac{g^2 m_H^2}{8m^2} (B^2 + H^2) \left(B^2 + H^2 + \frac{4m}{g} H \right).
\end{aligned}$$

Performing now an asymptotic analysis (that is, taking the Stückelberg field $B = 0$) it becomes

$$-\frac{g^2 m_H^2}{8m^2} \left(H^4 + \frac{4m}{g} H^3 \right).$$

4 Three MVBs

Let us now seek all gauge theories with *three* gauge fields. The only interesting Lie algebra entering the game is

$$\mathfrak{g} = \mathfrak{su}(2);$$

in this case obviously total antisymmetry implies the Jacobi identity.

The case $m_1 = m_2 = m_3 = 0$ is certainly possible, and then neither scalar Higgs nor Stückelberg fields are necessary.

The simplest of the *mass relations* we referred to in Section 2 is the following: if $f_{abc} \neq 0$ and $m_a = 0$, then necessarily $m_b = m_c$. We see at once from this that if $m_1 = 0$ must be $m_2 = m_3$: the case $m_1 = m_2 = 0, m_3 \neq 0$ is downright impossible.

The only other mass relation one needs to check to verify that models with two or three MVBs and one Higgs-like field are correct in our sense is

$$4C^2 m_b^2 m_a^2 = 2(m_a^2 + m_b^2) \sum_{d:m_d=0} (f_{abd})^2 + \sum_{k:m_k \neq 0} \frac{(f_{abk})^2}{m_k^2} [(m_a^2 + m_b^2 + m_k^2)^2 - 4(m_a^2 m_b^2 + m_k^4)]. \quad (25)$$

With $C^{-1} = \pm m_2$, the model with the mass pattern $m_2 = m_3 \neq 0, m_1 = 0$ passes muster.

If we assume that all masses are different from zero, then necessarily $m_1 = m_2 = m_3$. Indeed, equation (25) implies

$$4m_a^2 m_b^2 m_c^2 C^2 = [(m_a^2 + m_b^2 + m_c^2)^2 - 4(m_a^2 m_b^2 + m_c^4)]$$

where (a, b, c) is any permutation of $(1, 2, 3)$. Therefore,

$$m_1^2 m_2^2 + m_3^4 = m_2^2 m_3^2 + m_1^4 = m_3^2 m_1^2 + m_2^4.$$

This yields

$$\begin{aligned} (m_1^2 m_2^2 + m_3^4) - (m_2^2 m_3^2 + m_1^4) &= (m_3^2 - m_1^2)(m_1^2 - m_2^2 + m_3^2) = 0, \\ (m_2^2 m_3^2 + m_1^4) - (m_3^2 m_1^2 + m_2^4) &= (m_1^2 - m_2^2)(m_2^2 - m_3^2 + m_1^2) = 0, \end{aligned}$$

whose only all-positive solution is $m_1 = m_2 = m_3 =: m$; and then $4m^6 C^2 = m^4$ yields $C^{-1} = \pm 2m$.

Physically, the two cases just examined correspond respectively to the Georgi–Glashow model of electroweak interactions without neutral currents; and to the $\mathfrak{su}(2)$ Higgs–Kibble model. Reference [13] claims that more than one Higgs-like particle for the $\mathfrak{su}(2)$ Higgs–Kibble model is not allowed. It is well known that the first mass pattern obtained here is arrived at by SSB when the Higgs sector is chosen to be a $SU(2)$ isovector; and the second one when it is a complex doublet. But in our derivation SSB played no role.

5 The Weinberg–Salam model within CGI

Scharf and coworkers (see references in the introduction) followed a “deductive” approach to the SM, with the only assumption that m_1, m_2, m_3 are all positive, plus existence of the photon, that is, $m_4 = 0$. There is no point in repeating that. Suffice to say that a structure constant like f_{124} is found to

be non-zero, thus $m_1 = m_2$; and also the mass constraints imply $m_3 > m_1$. Defining

$$\cos \theta_W := m_1/m_3,$$

it is possible now to take for the non-zero structure constants

$$|f_{123}| = \cos \theta_W \quad \text{and} \quad |f_{124}| = \sin \theta_W.$$

With this, simply bringing (4) together with equations with (5), (6) and (7), one retrieves the boson part of the SM Lagrangian, as given for example in [23].

Thus it appears that the ordinary version of the Higgs sector for the gauge group $SU(2) \times U(1) \simeq U(2)$ is “chosen” by CGI. Of course, one can argue for it from other considerations within the SSB framework, or refer to experiment. We comment in the final discussion on the problem of determining which patterns of broken symmetry are allowed in CGI for general gauge groups.

5.1 Coupling to matter

Things stay interesting when considering the fermion sector. The basic interaction between carriers and matter in a gauge theory is of the form

$$g(b^a A_{a\mu} \bar{\psi} \gamma^\mu \psi + b'^a A_{a\mu} \bar{\psi} \gamma^\mu \gamma^5 \psi),$$

with $\bar{\psi}$ the Dirac adjoint spinor and b, b' appropriate coefficients. In dealing with the SM our fermions are the known ones, fulfilling as free fields the Dirac equation: we do not assume chiral fermions *ab initio*. Their gauge variation is taken to be zero. Thus for the SM one makes the Ansatz

$$\begin{aligned} T_1^F = & b_1 W_\mu^+ \bar{e} \gamma^\mu \nu + b'_1 W_\mu^+ \bar{e} \gamma^\mu \gamma^5 \nu + b_2 W_\mu^- \bar{\nu} \gamma^\mu e + b'_2 W_\mu^- \bar{\nu} \gamma^\mu \gamma^5 e \\ & + b_3 Z_\mu \bar{e} \gamma^\mu e + b'_3 Z_\mu \bar{e} \gamma^\mu \gamma^5 e + b_4 Z_\mu \bar{\nu} \gamma^\mu \nu + b'_4 Z_\mu \bar{\nu} \gamma^\mu \gamma^5 \nu \\ & + b_5 A_\mu \bar{e} \gamma^\mu e + b'_5 A_\mu \bar{e} \gamma^\mu \gamma^5 e + b_6 A_\mu \bar{\nu} \gamma^\mu \nu + b'_6 A_\mu \bar{\nu} \gamma^\mu \gamma^5 \nu \\ & + c_1 B^+ \bar{e} \nu + c'_1 B^+ \bar{e} \gamma^5 \nu + c_2 B^- \bar{\nu} e + c'_2 B^- \bar{\nu} \gamma^5 e \\ & + c_3 B_Z \bar{e} e + c'_3 B_Z \bar{e} \gamma^5 e + c_4 B_Z \bar{\nu} \nu + c'_4 B_Z \bar{\nu} \gamma^5 \nu \\ & + c_5 H \bar{\nu} \nu + c'_5 H \bar{\nu} \gamma^5 \nu + c_6 H \bar{e} e + c'_6 H \bar{e} \gamma^5 e. \end{aligned} \tag{26}$$

Here e stands for an electron, muon or neutrino or a (suitable combination of) quarks d, s, b ; and ν for the neutrinos or the quarks u, c, t ; the charge difference is always minus one. For instance in the “vertex” $W_\mu^+ \bar{e} \gamma^\mu \nu$ a “positron” exchanges a W^+ boson and becomes a “neutrino”. Charge is conserved in each term.

The method to determine the coefficients in (26) *remains the same*; only, it is simpler in practice. We limit ourselves to a few remarks. The direct equation

$$sT_1^F = i\partial \cdot Q_1^F$$

already allows to determine $c_1, c'_1, c_2, c'_2, c_3, c'_3, c_4, c'_4$, as well as the vanishing of b'_5 and b'_6 , assuming nonvanishing fermion masses. (For ν representing a true neutrino, we expect the term with coefficient b'_6 to vanish anyway, since the photon should not couple to uncharged particles. The same is true for b_6 .) Thus the photon has no axial-vector couplings, “because” there is no Stückelberg field for it, that is, because it is massless. The reader will have no trouble in finding the explicit form of Q_1^F , that can be checked with [15, Eq. 4.7.4]. At second order, one needs to take into account the interplay of contractions between Q_1 and T_1^F , as well as the “purely fermionic” ones between Q_1^F and T_1^F . There are no contractions between Q_1^F and T_1 , since the former does not contain derivatives. Also, *no new normalization terms* with fermionic fields may be forthcoming in sN_2 or $\partial_x \cdot N_{2/1}, \partial_y \cdot N_{2/2}$, since a term $\sim \varphi_1 \varphi_2 \bar{\psi} \psi \delta$ would be nonrenormalizable by power counting: the only way to cancel local terms is that the coefficient of every generated Wick monomial add up to zero.

At the end of the day, the physical Higgs couplings are proportional to the mass, and *chirality* of the interactions is a *consequence* of CGI [11, 12]. For leptons it yields:

$$\begin{aligned} T_1^F = & \frac{1}{2\sqrt{2}} W_\mu^+ \bar{e} \gamma^\mu (1 \pm \gamma_5) \nu + \frac{1}{2\sqrt{2}} W_\mu^- \bar{\nu} \gamma^\mu (1 \pm \gamma_5) e + \frac{1}{4 \cos \theta_W} Z_\mu \bar{e} \gamma^\mu (1 \pm \gamma_5) e \\ & - \sin \theta_W \tan \theta_W Z_\mu \bar{e} \gamma^\mu e - \frac{1}{4 \cos \theta_W} Z_\mu \bar{\nu} \gamma^\mu (1 \pm \gamma_5) \nu + \sin \theta_W A_\mu \bar{e} \gamma^\mu e \\ & + i \frac{m_e - m_\nu}{2\sqrt{2} m_W} B^+ \bar{e} \nu \pm i \frac{m_e + m_\nu}{2\sqrt{2} m_W} B^+ \bar{e} \gamma^5 \nu - i \frac{m_e - m_\nu}{2\sqrt{2} m_W} B^- \bar{\nu} e \\ & \pm i \frac{m_e + m_\nu}{2\sqrt{2} m_W} B^- \bar{\nu} \gamma^5 e \pm i \frac{m_e}{2 m_W} B_Z \bar{e} e \pm i \frac{m_\nu}{2 m_W} B_Z \bar{e} \gamma^5 \\ & + \frac{m_\nu}{2 m_W} H \bar{\nu} \nu + \frac{m_e}{2 m_W} H \bar{e} e, \end{aligned}$$

as it should. In summary we have recovered the SM, with its rationale upside-down.

6 Discussion

People define $e = g \sin \theta_W$; $g' = g \tan \theta_W$. Therefore,

$$\sec \theta_W = \frac{\sqrt{g^2 + g'^2}}{g},$$

and selecting the chiral projector and apart from the standard factors, the effective coupling of the term in $W_\mu^+ \bar{e} \gamma^\mu \nu$ and conjugate is g ; that of the term $A_\mu \bar{e} \gamma^\mu e$ is e ; that of the term $Z_\mu \bar{\nu} \gamma^\mu \nu$ is $-\sqrt{g^2 + g'^2}/2g$; and so on. Thus one can artfully write things as if g, g' are two different coupling constant associated to the emerging representation of the gauge group. But we have seen that the coefficients come from the pattern of masses, which in our viewpoint is fixed by nature. In order to bring home the point, let us make the *Gedankenexperiment* of building the SM from the Georgi–Glashow model, by adding a vector boson, sitting on an invariant abelian subgroup. Implicitly we allow for two different coupling constants (plus mixing of the old photon and the new MVB). But in that case there is no reason for $m_Z > m_W$. It is more natural to assume that the SM stems from the Higgs–Kibble model, keeping one coupling constant, whereby two of the three masses are moderately “pulled down” by mixing with the new photon. This goes to the heart of the experimental situation; other weak isospin values do not enter the game. In other words, no support comes from our quarter to the idea that the SM as it stands is “imperfectly unified”. The argument is bolstered by the fact that the true group of the electroweak interaction is $U(2)$, not $SU(2) \times U(1)$.²

In usual presentations of the SM the $U(2)$ symmetry is said to be “broken”, among other reasons, because there is only one conserved quantity, electric charge, instead of four. In CGI the interaction appears to respect the $U(2)$ symmetry. But of course symmetry is broken already at the level of the free Lagrangian, due to different masses (the residual symmetry $m_1 = m_2$ goes in hand with electric charge conservation). This is to say that not all bases of the Lie algebra are equivalent, since there is a natural basis dictated by the pattern of masses. The role of the mass constraints is precisely to pick out this basis.³

²To our knowledge, this was noticed first in [24].

³In the early seventies, speculations on fermionic patterns of masses from SSB were rife—see for instance [25]. They have been since all but abandoned. They might perhaps become a respectable subject of study again in CGI. By now we may only say that differences between fermion masses are related to differences between boson masses in that (disregarding family mixing) models in which all bosons share the same mass would entail identity of all fermion masses as well.

Let us recapitulate. CGI is a tool for the actual construction of Lagrangians. We limited ourselves to polynomial couplings. At first order in the coupling constant, CGI fixes some of the couplings of the vector bosons and ghost (fermionic and bosonic) fields. At second order, it requires additional quartic couplings, as well as some extra ingredient, which here is made out of physical scalars or Higgs-like fields. Third-order invariance goes on to fix the remaining couplings of the Higgs-like fields. One obtains in that way potentials of the symmetry-breaking kind, although SSB does not enter the picture. Ockham’s razor, already invoked in Section 2 in relation with the number of Higgs fields, seems even more pertinent here.

On the historical side, it is difficult to imagine the development of electroweak unification during the sixties without the SSB crutch. Massive vector bosons were beyond the pale then. The only contemporaneous article (still instructive today) I know of, willing and eager to start from them as fundamental entities is [26]; it did not have enough impact. Around ten years later, after the invention of SSB, cogent arguments based on tree unitarity—see [27] and references therein—weighed in favour of the phenomenological outcome of gauge theories with broken symmetry, plus abelian mass terms for invariant abelian subgroups. This is basically what CGI constructs.

Since they lead to the same phenomenological Lagrangian, there seems to be no way as yet—within ordinary particle physics, at least—to distinguish between the SM as presented in textbooks and its causal version. This is good, because it shows that CGI is solidly anchored in physics.

It is also bad: “a difference, to be a difference, has to make a difference”. Still, a constructive CGI program was in principle attractive because the apparent severity of the constraints on the masses of the gauge fields. Ambauen and Scharf [28] argued that the $SU(5)$ grand unification model by Georgi and Glashow with its standard pattern of Higgs fields [29, Chap. 18], is not causally gauge invariant; and the situation in this respect for a while was murky. However, a systematic comparison between CGI and the general theory of broken local symmetries [30] has been performed recently [31], and the contention of [28] that there might be contradiction between causal gauge invariance and some grand unified models has been laid to rest.

Acknowledgments

I am most grateful for discussions to Luis J. Boya, Florian Scheck and Joseph C. Várilly. Special thanks are due to Michael Dütsch, who patiently explained to me aspects of the gauge principle according to the Zürich school. I acknowledge support from CICyT, Spain, through grant FIS2005–02309.

References

- [1] I. J. R. Aitchison and A. J. G. Hey, *Gauge theories in particle physics: QCD and the electroweak theory*, IOP Publishing, Bristol, 2004.
- [2] C. Burgess and G. Moore, *The Standard Model: a primer*, Cambridge University Press, Cambridge, 2007.
- [3] M. J. G. Veltman, Phys. Rev. Lett. **34** (1975) 777.
- [4] H. Cheng and E-C. Tsai, Phys. Rev. D **40** (1989) 1246.
- [5] M. J. G. Veltman, Rev. Mod. Phys. **72** (2000) 341.
- [6] J. Earman, Philos. Sci. **71** (2004) 1227.
- [7] H. Lyre, Intl. Studies Philos. Sci. **22** (2008) 119.
- [8] H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A **19** (2004) 3265.
- [9] J. M. Gracia-Bondía, “BRS invariance for massive boson fields”, to appear in the Proceedings of the Summer School “Geometrical and topological methods for quantum field theory”, Cambridge University Press, Cambridge, 2010; hep-th/0808.2853.
- [10] W. Kilian, *Electroweak symmetry breaking: the bottom-up approach*, Springer, New York, 2003.
- [11] M. Dütsch and G. Scharf, Ann. Phys. (Leipzig) **8** (1999) 359.
- [12] A. Aste, G. Scharf and M. Dütsch, Ann. Phys. (Leipzig) **8** (1999) 389.
- [13] M. Dütsch and B. Schroer, J. Phys. A **33** (2000) 4317.
- [14] D. R. Grigore, J. Phys. A **33** (2000) 8443.
- [15] G. Scharf, *Quantum gauge theories. A true ghost story*, Wiley, New York, 2001.
- [16] T. Hurth and K. Skenderis, Nucl. Phys. B **541** (1999) 566.
- [17] T. Hurth and K. Skenderis, in *New developments in quantum field theory*, P. Breitenlohner, D. Maison and J. Wess, eds., Springer, Berlin, 2000; pp. 86–105.
- [18] H. Epstein and V. Glaser, Ann. Inst. Henri Poincaré **XIXA** (1973) 211.

- [19] L. Alvarez-Gaumé and L. Baulieu, Nucl. Phys. B **212** (1983) 255.
- [20] R. Stora, “Local gauge groups in quantum field theory: perturbative gauge theories”, talk given at the workshop “Local quantum physics”, Erwin Schrödinger Institute, Vienna, 1997.
- [21] M. Dütsch, private communication.
- [22] M. Dütsch, Ann. Phys. (Leipzig) **14** (2005) 438.
- [23] M. Veltman, *Diagrammatica*, Cambridge University Press, Cambridge, 1994.
- [24] F. Scheck, *Leptons, hadrons and nuclei*, North-Holland, Amsterdam, 1983.
- [25] H. Georgi and S. L. Glashow, Phys. Rev. D **6** (1972) 2977.
- [26] V. I. Ogievetskij and I. V. Polubarinov, Ann. Phys. (New York) **25** (1963) 358.
- [27] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. D **10** (1974) 1145.
- [28] M. Ambauen and G. Scharf, “Violation of quantum gauge invariance in Georgi–Glashow $SU(5)$ ”, hep-th/0409062.
- [29] H. Georgi, *Lie algebras in particle physics*, Westview Press, Boulder, 1999.
- [30] Ling-Fong Li, Phys. Rev. D **9** (1974) 1723.
- [31] M. Dütsch, J. M. Gracia-Bondía, F. Scheck and J. C. Várilly, “Quantum gauge models without classical Higgs mechanism”, hep-th/1001.0932.